Part 3: Safety and liveness
Safety vs. liveness

Safety: something “bad” will never happen
Liveness: something “good” will happen (but we don’t know when)
Safety vs. liveness for sequential programs

Safety: the program will never produce a wrong result ("partial correctness")

Liveness: the program will produce a result ("termination")
Safety vs. liveness for sequential programs

Safety: the program will never produce a wrong result (“partial correctness”)

Liveness: the program will produce a result (“termination”)

Safety vs. liveness for state-transition graphs

**Safety:** those properties whose violation always has a finite witness
("if something bad happens on an infinite run, then it happens already on some finite prefix")

**Liveness:** those properties whose violation never has a finite witness
("no matter what happens along a finite run, something good could still happen later")
Safety: the properties that can be checked on finite executions

Liveness: the properties that cannot be checked on finite executions

(they need to be checked on infinite executions)
Example: Mutual exclusion

It cannot happen that both processes are in their critical sections simultaneously.
Example: Mutual exclusion

It cannot happen that both processes are in their critical sections simultaneously.

Safety
Example: Bounded overtaking

Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter.
Example: Bounded overtaking

Whenever process P1 wants to enter the critical section, then process P2 gets to enter at most once before process P1 gets to enter.

Safety
Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.
Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

Liveness
Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

Liveness
LTL (Linear Temporal Logic)

- safety & liveness
- linear time

[Pnueli 1977; Lichtenstein & Pnueli 1982]
LTL Syntax

\[ \varphi ::= a \mid \varphi \land \varphi \mid \neg \varphi \mid \Diamond \varphi \mid \varphi \cup \varphi \]
LTL Model

infinite trace $t = t_0 \ t_1 \ t_2 \ ...$
(sequense of observations)
Run: \( q_1 \rightarrow q_3 \rightarrow q_1 \rightarrow q_3 \rightarrow q_1 \rightarrow q_2 \rightarrow q_2 \rightarrow \)

Trace: \( a \rightarrow b \rightarrow a \rightarrow b \rightarrow a \rightarrow a,b \rightarrow a,b \rightarrow \)
Language of **deadlock-free** state-transition graph $K$ at state $q$:

$L(K,q) = \text{set of infinite traces of } K \text{ starting at } q$

$(K,q) |= \forall \varphi$ iff for all $t \in L(K,q)$, $t |= \varphi$

$(K,q) |= \exists \varphi$ iff exists $t \in L(K,q)$, $t |= \varphi$
LTL Semantics

\[ t \models a \iff a \in t_0 \]

\[ t \models \varphi \land \psi \iff t \models \varphi \text{ and } t \models \psi \]

\[ t \models \neg \varphi \iff \text{not } t \models \varphi \]

\[ t \models \Diamond \varphi \iff t_1 t_2 \ldots \models \varphi \]

\[ t \models \varphi \mathbin{U} \psi \iff \exists \ n \geq 0 \text{ s.t.} \]
\[ \begin{align*}
1. & \quad \text{for all } 0 \leq i < n, \ t_i \ t_{i+1} \ldots \models \varphi \\
2. & \quad t_n \ t_{n+1} \ldots \models \psi
\end{align*} \]

\[(K,q) \models \forall \varphi \iff \neg (K,q) \models \exists \neg \varphi \]
Defined modalities

\[ \begin{align*}
\bigcirc & \quad \text{next} \\
\bigcup & \quad \text{until} \\
\Diamond \varphi & = \text{true} \bigcup \varphi \\
\square \varphi & = \neg \Diamond \neg \varphi \\
\varphi \mathcal{W} \psi & = (\varphi \bigcup \psi) \lor \square \varphi \\
\end{align*} \]
Important properties

Invariance

\( \Box \ a \)

\( \Box \ \neg (pc1=\text{in} \land pc2=\text{in}) \)

Sequencing

\( a \ W \ b \ W \ c \ W \ d \)

\( \Box \ (pc1=\text{req} \implies \)

\( (pc2\neq\text{in}) \ W (pc2=\text{in}) \ W (pc2\neq\text{in}) \ W (pc1=\text{in})) \)

Response

\( \Box \ (a \implies \Diamond \ b) \)

\( \Box \ (pc1=\text{req} \implies \Diamond (pc1=\text{in})) \)
Composed modalities

□◊ a infinitely often a
◊□ a almost always a
Example: Starvation freedom

Whenever process P1 wants to enter the critical section, provided process P2 never stays in the critical section forever, P1 gets to enter eventually.

□ ◇ (pc2=in ⇒ ◇ (pc2=out)) ⇒
□ (pc1=req ⇒ ◇ (pc1=in))
State-transition graph

\( Q \) \hspace{1cm} \text{set of states} \hspace{1cm} \{q_1, q_2, q_3\}

\( A \) \hspace{1cm} \text{set of atomic observations} \hspace{1cm} \{a, b\}

\( \rightarrow \subseteq Q \times Q \) \hspace{1cm} \text{transition relation} \hspace{1cm} q_1 \rightarrow q_2

\([\cdot] : Q \rightarrow 2^A\) \hspace{1cm} \text{observation function} \hspace{1cm} [q_1] = \{a\}
(K,q) \models \forall \varphi

Tableau construction (Vardi-Wolper)

(K', q', BA) \text{ where } BA \subseteq K'

Is there an infinite path starting from q' that hits BA infinitely often?

Is there a path from q' to p \in BA such that p is a member of a strongly connected component of K'?
dfs(s) {
    add s to dfsTable
    for each successor t of s
        if (t \notin dfsTable) then dfs(t)
    if (s \in BA) then { seed := s; ndfs(s) }
}

ndfs(s) {
    add s to ndfsTable
    for each successor t of s
        if (t \notin ndfsTable) then ndfs(t)
        else if (t = seed) then report error
}